ESTIMATION OF SOLAR MASS AND DISTANCE, FROM THE EQUILIBRIUM OF ELASTIC AND GRAVITATING FORCES.

By Pliny Earle Chase.

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The lightest and most elastic gas must be kept in mean position, under conditions of equilibrium between the forces of gaseous expansion and of virtual fall towards the centres of the Sun and the Earth. In cases of explosive disturbance, the character and range of the resulting oscillations must depend on those conditions of equilibrium.

In order to ascertain the approximate ratio of terrestrial to solar gravity, let us suppose the mass of the Earth concentrated in a single point, c, at its centre of gravity.

Let x = distance from c at which the satellite and planetary velocities would be equal.

$\zeta = 2x$; $d = 2\zeta$; r = Earth's radius.

g, g^0, g^1 = force of gravity at ζ, r, and x, respectively.

v = orbital velocity of earth, or satellite velocity at x.

η = distance of Sun from Earth.

μ = mass (Sun ÷ Earth).

T^d = time of actual fall through a diameter d, or time of orbital revolution at the mean distance x.

τ^δ = time of actual fall through a diameter δ, or time of virtual fall through a distance = δ at ζ.

$\tau_1{}^\delta$ = time of virtual fall through a distance = δ, at r.

Then by the laws of gravitation, we have the proportion

$$T^d : \tau^\delta :: d^{\frac{3}{2}} : \delta^{\frac{3}{2}} :: \tfrac{\pi}{2}\sqrt{\frac{d}{g}} : \sqrt{\frac{2\delta}{g}}$$

$$1 : \left[\frac{\delta}{d}\right]^{\frac{3}{2}} :: \frac{\pi}{\sqrt{8}} : \left[\frac{\delta}{d}\right]^{\frac{1}{2}}$$

$$d : \delta :: \pi : \sqrt{8}$$

$$T^d \propto \tau^\delta \propto \sqrt{\frac{1}{g}}$$

The six most recent experimenters upon the explosive force of hydrogen, have obtained results with a limiting variation of about $3\frac{1}{2}$ per cent. from the mean. Four of the experimenters agree within an extreme difference of less than two-fifths of one per cent., the mean of their results differing from the general mean by less than one-fifth of one per cent. This agreement is much closer than any hitherto obtained by astronomical observations.

According to the experiments referred to, the explosive force of H_2O may be represented by a virtual fall through a mean δ of 1017.01 miles, in a $\tau_1{}^\delta = \sqrt{\frac{2\delta}{g^0}}$ = 578.5 seconds. The best approximate values of δ are those of

Dulong.................	D	1015.97	miles.
Hess.....................	H	1017.40	"
Crawford...............	C	1052.73	"
Grassi....................	G	1013.72	"
Favre and Silbermann...	F	1013.60	"
Adams..................	A	988.63	"
Mean of D, H, G, F.........................		1015.18	"
General Mean..................................		1017.01	"

If we assume the correctness of the general mean, $d = \delta \times \sqrt{\frac{\pi}{8}} =$ 1129.61 ; $\tau^{\delta} = \tau_1^{\delta} \times \sqrt{\frac{g^0}{g}} = 578.5 \times \frac{1129.61}{7925.64} = 82.45$ seconds ; $x = d \div 4 = 282.4$; $T^d = \tau^{\delta} \div \left[\frac{\delta}{d}\right]^{\frac{3}{2}} = 96.515$ seconds ; $v = 2 \pi x \div T^d =$ 18.3844 miles ; $\eta = v \times 1$ year (in seconds) $\div 2 \pi = 92,338,000$ miles ; $\mu = \eta \div x = 326,980$.

This approximation is subject to correction for possible imperfect elasticity of hydrogen, æthereal resistance, and orbital eccentricity. From various considerations I am inclined to believe that the aggregate corrections for the value of η, cannot exceed to one and a-half per cent. of the above amount.

NOTE ON PLANETO-TAXIS.

By Pliny Earle Chase.

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I am not aware that any reason has ever been assigned for the planetary harmony which is formulated in "Bode's Law," or that any attempt has been made to show that the failure of the analogy, in the case of Neptune, is really only one of those apparent exceptions which serve to establish general rules on a firmer basis.

The many evidences which I have already adduced, of simple relationships between planetary positions and centres of oscillation, seem to furnish the needed data for verifying the law, as a simple and natural resultant of equilibrating forces, and not a mere accidental coincidence. If a nebulous mass were set in rotation, each of its equatorial radii might be regarded as a simple pendulum, with a tendency to vibrate in the same time as its centre of oscillation, which tendency might be expected to produce an aggregation at that centre.

If we start from the circularly divided radius next within the orbit of Mercury, $\left(\frac{\pi}{32} r = .0982\right)$, and add multiples of the next following radius $\left(\frac{\pi^2}{32} r = .3085\right)$, we may form the first series (A) in the following table,